

Virtual Formula 2022 Rules & Regulations

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1. Competition Overview

For more than ten years, VI-grade encouraged Formula SAE and Formula Students time to learn professional vehicle dynamics simulation tools by instituting a simulation-based competition called Virtual Formula. The focus in the early years was to optimize a baseline vehicle model running it through a selection of FSAE/FS competition typical events.

As professional vehicle dynamics engineering has evolved towards a more intense usage of realtime simulation, VI-grade decided to advance the format of the Virtual Formula competition to include Driver-in-the-Loop simulation.

1.1. Competition Description

Teams competing in the VI-grade 2022 Virtual Formula will endeavor to create in VI-CarRealTime a digital twin of their 2022 Formula SAE or Formula Student entry.

The Virtual Formula competition will be conducted in two phases. Phase 1 will focus on exercising the team's VI-CarRealTime model submission through the following offline dynamic events to assess the validity of the models and model compliance with the virtual formula rules. Validation maneuvers will be the same as defined in the FSAE/FS rules:

- Skidpad
- Acceleration
- Autocross
- Endurance

Once models and supporting data have been assessed and validated to run in offline simulation, the team will be qualified for the next phase of the competition.

During phase 2, qualified teams will run their VI-CarRealTime model through the Endurance course with a Driver-in-the-Loop using a desktop simulator based on the VI-DriveSim software.

The final ranking of the competition will be determined by the overall race time, minus penalties (cutting corners, hitting cones, setup irregularities).

1.2. Objective

Teams are requested to submit a VI-CarRealTime model intended to represent the team's 2022 entry and meant to be consistent with the spirit of the Formula SAE rules section:

- V Vehicle Requirements
- F Chassis and Structural
- T Technical Aspects
- IC Internal Combustion
- EV Electric Vehicle
- D Dynamic Events

or of the Formula Student Germany rules section:

T – General Technical Requirements (omit autonomous sections)

CV – Internal Combustion Engine Vehicles

EV - Electric Vehicles

D – Dynamic Events (omit Driverless sections)

1.3. Classes

The competition is separated into two classes (Internal Combustion Engine (CV) and Electric Vehicle (EV)).

1.4. Events

The following table describes the objective of the competition events.

Event	Objective	Phase of competition
Skidpad	Minimize time to complete the course	Phase 1 validation / non-scored
Acceleration	Minimize time to complete the course	Phase 1 validation / non-scored
Autocross	Minimize time to complete the course	Phase 1 validation / non-scored
Endurance	Minimize time to complete the course	Phase 2 driving / scored (See paragraph 3.4)
Efficiency	Maximize energy efficiency	Phase 2 driving / scored (See paragraph 3.5)

Table 1: Event Objectives

2. Vehicle Model Rules

The following section describes how teams may construct their VI-CarRealTime vehicle model.

2.1. Legal Parameters

VI-CarRealTime models are reduced order models that represent systems or groups of components as the aggregate behaviors of those systems or component groups. This section will discuss the primary subsystems of a VI-CarRealTime vehicle model, and the flexibility or limits teams have in defining their model.

The mandatory requirement is that the digital twin created within VI-CarRealTime will have to be consistent with the section 1.2 Objective "Spirit of the rules", that is to be coherent with the reallife FSAE rule book, for the sake of the exercise and the competition.

Examples would be FSAE rules V.1.1. Open Wheel configuration, T.7.4 Aero device plan view measurements, IC.4.3 maximum restrictor diameter, etc.

In addition to the limitations of section 1.2 Objective, teams have the following options and limits (1 through 2.2.6) imposed on their vehicle design intent converted into the VI-CarRealTime model.

2.2. Front and Rear suspension

Kinematics and compliance will be modeled with VI-grade VI-CarRealTime using the VI-SuspensionGen configurations and inputs.

- Springs VI-CarRealTime input data limit to linear rates unless test data is available.
- Dampers VI-CarRealTime input data limited to published curves by suppliers, unless test data is available. Simulink damper models are allowed only with review and agreement with Virtual Formula rules representatives.

2.2.1. Tires

Teams will have the option to use a selection of 4 example tire models or to generate Pac2002 or MFtyre 6.2 tire models from TTC data.

Teams generating their own tire models must show the fit of the tire models to test data. Since the peak FX/FZ and peak FY/FZ of the TTC data very often does not correspond to real world road surface frictional values, VI-grade dictates that all tire models will have to be scaled such that peak FX/FZ and peak FY/FZ values at FZ=200N do not exceed 1.4 for all combinations of slip angle and inclination.

2.2.2. Powertrain IC

Teams will have the option to use one of the three provided example engine power maps or physical test data. The example engine maps include a 400cc single cylinder, a 600cc four cylinder, and a 600cc 4-cylinder turbo charged.

Gear ratios must be those from the engine's published specifications and the team's vehicle design intended sprocket final drive ratio.

VI-CarRealTime Differential input data must be representative of the team's vehicle design intent (open, limited slip, spool, etc.)

2.2.3. Powertrain EV

Teams entering an electric vehicle must comply with the functional requirements of the 2021 FSAE or FS rules. Satisfy the "functional requirements" means that the data input into VI-CarRealTime is representative of a design that complies with FSAE/FS rules and could pass FSAE/FS on-site technical inspection. Examples of a functional requirements are EV.4.1.1 Max. Power not to exceed 80kW and EV.4.1.2 max. Voltage not to exceed 600VDC.

Teams competing with an electric vehicle must activate and populate in VI-CarRealTime the "Battery" section under "Driveline Layout" for the powertrain subsystem see Figure 1. More details of the battery model and variables are available in the appendix (Figure 4, Figure 5, and Figure 6).



Figure 1: Accumulator representation in VI-CarRealTime

Teams running a vehicle with a rear-wheel-drive single motor with a differential must use the configuration in



Figure 2.



Figure 2: RWD single motor configuration

Teams running a vehicle with one motor per wheel without a differential must use the configuration in Figure 3 with the appropriate wheel motors selected for the team's design intent.



Figure 3: Multiple motor without differential configuration

2.2.4. Body CG and Inertia

Teams must include documentation of the vehicle weight, CG, and Inertia. The documentation can include calculations, CAD data, previous year technical inspection sticker weight and distribution, or measured data.

2.2.5. Aerodynamics

Aerodynamic maps will be limited to the VI-CarRealTime "standard" *.aer property file format. The data is inputted as three tables:

- Front downforce vs. rear ride height vs. front ride height.
- Rear downforce vs. rear ride height vs. front ride height.
- Drag force vs rear ride height vs. front ride height.

Aerodynamic forces must be calculated at the following conditions:

- reference_velocity <kph> = 60.0
- air_density <kgpm3> = 1.2 \$[kg/m^3]
- reference_density = 1.0

2.2.6. Controllers

Teams may not use the native controllers in VI-CarRealTime (traction control or ABS). Teams can include Simulink based controllers if the team's design intent is to compete in FSAE / FS with those programs on the vehicle entry.

3. Events

The following section describes non-scored events that will be used to evaluate model quality and compliance to rules requirements. The same vehicle model and setup will be used across all events.

3.1. Skidpad

The skidpad course consists of two pairs of concentric circles in a figure of eight pattern. The circles have a radius of 9.125m measured from the centerline of the road. The road has a constant width of 3m. At the outside edges of the road, there is a kerb of 100mm height that is intended to penalize vehicles exploiting track limits.

The following event settings must be set:

- Path File: VF2022_Skidpad.drd
- Road Data File: VF2022_Skidpad.rdf
- Road Graphic File: VF2022_Skidpad.obj
- Flying lap: off
- User Defined Location: on (0, 0, Automatic Z Location on, 0, 0, 0)
- Laps: on, min = 1, max = 1

3.2. Acceleration

The acceleration course is a straight line with length of 75m.

The following event settings must be set:

- VI Driver Input File: VF2022_Acceleration.vdf
- Road Data File: VF2022_Acceleration.rdf
- Road Graphic File: VF2022_Acceleration.obj
- User Defined Location: on (0, 0, Automatic Z Location on, 0, 0, 0)

3.3. Autocross

The autocross course is a closed circuit. The road has a constant width of 3m. At the outside edges of the road, there is a kerb of 100mm height that is intended to penalize vehicles exploiting track limits.

The following event settings must be set:

- Path File: VF2022_RaceTrack.drd
- Road Data File: VF2022_RaceTrack.rdf
- Road Graphic File: VF2022_RaceTrack.obj
- Flying lap: off
- Initial Velocity: 1000
- User Defined Location: on (0, 0, Automatic Z Location on, 0, 0, 0)
- Laps: on, min = 1, max = 1

3.4. Endurance

The endurance course is a closed circuit. The road has a constant width of 3m. At the outside edges of the road, there is a kerb of 100mm height that is intended to penalize vehicles exploiting track limits.

The following event settings must be set:

- Path File: VF2022_RaceTrack.drd
- Road Data File: VF2022_RaceTrack.rdf
- Road Graphic File: VF2022_RaceTrack.obj
- Flying lap: on
- User Defined Location: on (0, 0, Automatic Z Location on, 0, 0, 0)
- Laps: on, min = 1, max = 1

The following equation describes how the endurance score is determined.

$$EnduranceScore = 300 * \left(\frac{Tmax/Tyour - 1}{Tmax/Tmin - 1}\right)$$

Where:

- T_{your} is the time your team took to complete the require course laps.
- T_{\min} is the best time any team took to complete the require course laps.
- T_{max} is the best time of all the teams multiplied by 1.45.
- Minimum score is 0

3.5. Efficiency CV

Energy efficiency is measured during the endurance event. Teams who score zero for endurance will also score 0 for efficiency.

Energy sources (fuel type or electricity) will be converted to kg of CO₂. Conversion factors are listed in the table below.

Gasoline / Petrol	2.31	kg of CO ₂ per liter
E85	1.65	kg of CO ₂ per liter
Electric	0.65	kg of CO ₂ per kWh

Table 2: Efficiency conversion fa	factors
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The following equation describes how the efficiency score is determined.

$$EfficiencyFactor = \left(\frac{Tmin}{Tyour}\right) * \left(\frac{CO2min}{CO2your}\right)$$

Where:

- CO_{2your} is the team's efficiency factor
- CO_{2min} is the highest efficiency factor of all teams
- CO_{2max} is the zero-point threshold derived from 60.6 kg CO₂ / 100Km and the total distance traveled during the endurance event (course distance time laps).
- T_{your} is the time your team took to complete the require course laps.
- T_{\min} is the best time any team took to complete the require course laps.
- T_{max} is the best time of all the teams multiplied by 1.45.
- Minimum score is 0

The following equation describes how the efficiency factor is determined.

$$EfficiencyScore = 100 * (\frac{\frac{EfficiencyFactorMin}{EfficiencyFactorYour} - 1}{\frac{EfficiencyFactorMin}{EfficiencyFactorMin} - 1})$$

Where:

- EfficiencyFactorMin is calculated using CO_{2your} = CO_{2min}, and T_{your} = T_{max}
- *EfficiencyFactorMax* is highest EfficiencyFactor of all teams

3.6. Final Ranking

The final ranking will be determined by the total score of: Final Score = Endurance Score + Efficiency Score Thus the final score will be at maximum 400 and at minimum 0.

3.7. Penalties

• Hitting a cone will subtract 5 points from the team's Endurance score

• Short cutting a portion of the course will subtract 10 points from the team's Endurance score.

4. Submissions

The following sections describes the content teams will submit for review and verification. Submissions are required to be sent by the deadline set for the Team's region. There will be three regions: Americas, Europe, and Asia. The submission deadlines will be posted on the VIgrade website (<u>https://www.vi-grade.com/en/vf2022#datestoremember</u>).

4.1. Requirements

The following is models and supporting document are required for phase 1 submission:

- Vehicle model database (*.cdb directory)
- Fingerprint file for the events as they were run (*.xml file)
- SuspensionGen kinematic and compliance (*.sgs file)
- Completed submission form
- Supporting test data or analysis:
 - o Engine torque curve (example: dyno run or data acquisition of acceleration run)
 - o Body mass and inertia (example: measurements or calculations based on CAD)
 - o Damper curve (example: shock dyno run or manufacturer published data)
 - o Aero map (example: CFD, wind tunnel data, or data acquisition with analysis)
 - o Tire model (example: Comparison of tire fit to TTC data and adjustment per Legal Parameters: Tires)

4.2. Late Submissions

No late submissions will be accepted.

4.3. Validated models for phase 2

VI-grade will review the models and validate that they comply with the requirements of the Legal Parameters section.

The verified models will be returned to the team in obfuscated format so that the model cannot be altered.

Teams will drive the obfuscated models in the Driver-in-the-Loop (online) competition.

5. Appendix

5.1. Powertrain IC engine Torque vs. RPM maps

4 cylinder 600cc

4 cylinder 600cc		
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RPM	0% throttle	100% throttle	Horsepower
0	0	0	0
250	-4,460	9731	0.3
500	-7,404	15111	1.1
750	-7,596	16023	1.7
1000	-7,736	16998	2.4
1250	-7,876	18010	3.2
1500	-8,016	18889	4.0
1750	-8,156	20079	4.9
2000	-8,296	21571	6.1
2250	-8,436	23027	7.3
2500	-8,576	24360	8.5
2750	-8,715	25769	9.9
3000	-8,856	27261	11.5
3250	-8,996	28674	13.1
3500	-9,135	30230	14.8
3750	-9,275	32015	16.8
4000	-9,415	33875	19.0
4250	-9,555	35046	20.9
4500	-9,695	35853	22.6
4750	-9,835	36963	24.6
5000	-9,975	38050	26.7
5250	-10,115	39060	28.8
5500	-10,255	40385	31.2
5750	-10,395	41982	33.9
6000	-10,535	43453	36.6
6250	-10,675	44817	39.3
6500	-10,814	46098	42.0
6750	-10,954	47127	44.6
7000	-11,095	48075	47.2
7250	-11,234	48552	49.4
7500	-11,374	48982	51.5
7750	-11,514	49585	53.9
8000	-11,654	50278	56.4
8250	-11,794	51030	59.1
8500	-11,934	51787	61.8
8750	-12,074	52550	64.5
9000	-12,214	52493	66.3
9250	-12,353	52364	68.0
9500	-12,493	52232	69.6

9750	-12,634	52027	71.2
10000	-12,773	51591	72.4
10250	-12,913	51147	73.6
10500	-13,053	50697	74.7
10750	-13,193	49805	75.1
11000	-13,333	48528	74.9
11250	-13,473	47235	74.6
11500	-13,613	45926	74.1
11750	-13,753	44885	74.0
12000	-13,894	43883	73.9

Single cylinder 400cc

Single	cylinder 400cc		
RPM	0% throttle	100% throttle	Horsepower
0	0	0	0
188	-2,867	12,656	0.3
375	-4,760	19,429	1.0
563	-4,883	20,365	1.6
750	-4,973	21,356	2.2
938	-5,063	22,369	2.9
1,125	-5,153	23,192	3.7
1,313	-5,243	24,372	4.5
1,500	-5,333	25,886	5.4
1,688	-5,423	27,318	6.5
1,875	-5,513	28,571	7.5
2,063	-5,603	29,880	8.6
2,250	-5,693	31,250	9.9
2,438	-5,783	32,498	11.1
2,625	-5,873	33,872	12.5
2,813	-5,963	35,466	14.0
3,000	-6,053	37,102	15.6
3,188	-6,143	37,950	17.0
3,375	-6,233	38,383	18.2
3,563	-6,322	39,125	19.6
3,750	-6,413	39,819	21.0
3,938	-6,503	40,414	22.3
4,125	-6,592	41,314	23.9
4,313	-6,682	42,462	25.7
4,500	-6,773	43,453	27.4
4,688	-6,862	44,310	29.1

4,875	-6,952	45,062	30.8
5,063	-7,042	45,547	32.4
5,250	-7,132	45,939	33.8
5,438	-7,222	45,869	35.0
5,625	-7,312	45,752	36.1
5,813	-7,402	45,792	37.3
6,000	-7,492	45,906	38.6
6,188	-7,582	46,065	40.0
6,375	-7,672	46,219	41.3
6,563	-7,762	46,367	42.7
6,750	-7,852	45,792	43.4
6,938	-7,941	45,161	44.0
7,125	-8,032	44,534	44.5
7,313	-8,122	43,855	45.0
7,500	-8,211	42,992	45.2
7,688	-8,301	42,137	45.5
7,875	-8,391	41,289	45.6
8,063	-8,481	40,100	45.4
8,250	-8,571	38,625	44.7
8,438	-8,661	37,165	44.0
8,625	-8,751	35,720	43.2
8,813	-8,841	34,509	42.7
9,000	-8,932	33,351	42.1

4 cylinder 600 turbo

4 cylind	ler 600 turbo		
RPM	0% throttle	100%	Horsepower
		throttle	
0	0	0	0
250	-4460	8,208	0.3
500	-7404	12,810	0.9
750	-7596	13,650	1.4
1000	-7736	14,550	2.0
1250	-7876	15,490	2.7
1500	-8016	16,322	3.4
1750	-8156	17,430	4.3
2000	-8296	18,810	5.3
2250	-8436	20,169	6.4
2500	-8576	21,429	7.5
2750	-8715	22,766	8.8

	3000	-8856	24,185	10.2
	3250	-8996	25,545	11.6
	3500	-9135	27,040	13.3
	3750	-9275	28,752	15.1
	4000	-9415	31,262	17.5
	4250	-9555	33,215	19.8
	4500	-9695	34,876	22.0
	4750	-9835	36,885	24.6
	5000	-9975	38,930	27.3
	5250	-10115	40,954	30.2
	5500	-10255	43,373	33.5
	5750	-10395	46,162	37.2
	6000	-10535	49,370	41.6
	6250	-10675	52,569	46.1
	6500	-10814	55,777	50.9
	6750	-10954	58,772	55.7
	7000	-11095	61,750	60.6
	7250	-11234	63,643	64.7
	7500	-11374	64,413	67.8
	7750	-11514	65,412	71.1
	8000	-11654	65,405	73.4
	8250	-11794	65,439	75.8
	8500	-11934	65,443	78.1
	8750	-12074	65,415	80.3
	9000	-12214	64,346	81.3
	9250	-12353	63,182	82.0
	9500	-12493	62,011	82.7
	9750	-12634	60,751	83.1
	10000	-12773	59,226	83.1
	10250	-12913	57,701	83.0
	10500	-13053	56,178	82.8
	10750	-13193	54,185	81.7
	11000	-13333	51,810	80.0
	11250	-13473	49,462	78.1
	11500	-13613	47,144	76.1
	11750	-13753	45,143	74.4
Ī	12000	-13894	43,217	72.8

5.2. Example aerodynamics map

\$-----MDI HEADER [MDI HEADER] FILE_TYPE = 'aer' $FILE_VERSION = 5.00$ FILE_FORMAT = 'ASCII' (COMMENTS) {comment_string} 'Sample Aero Data' \$-----------units [UNITS] (BASE) {length force angle mass time} mm' 'newton' 'degree' 'kg' 'sec' (USER) {unit_type length force angle mass time conversion} kph' 1 0 0 0 -1 0.0036 -----test_conditions \$-----[TEST_CONDITIONS] reference_velocity <kph> = 60 air_density <kgpm3> = 1.2 \$[kg/m^3] reference_density = 1 front_ride_height_min = 6.35 front_ride_height_max = 50 rear_ride_height_min = 6.35 rear_ride_height_max = 75 DRAG_ARM_HEIGHT_MIN = -50 DRAG_ARM_HEIGHT_MAX = 50 \$-----front_downforce [FRONT_DOWNFORCE] (Z_DATA) {rear_ride_height } 6.35 12.7 19.05 25.4 38.1 50.8 63.5 76.2 (XY_DATA) {front_ride_height downforce } 6.35 423.6 440.4 457.2 465.7 481.1 495.3 509.2 523.4 389.5 402.1 408.3 441.2 463.1 484.9 12.7 368.0 422.4 361.4 372.9 378.5 404.9 418.6 432.4 19.05 346.0 391.1 334.4 344.8 350.0 25.4 324.7 361.0 377.3 398.7 420.0 31.75 298.1 303.9 309.3 311.9 318.7 326.5 333.4 340.4 38.1 272.6 274.8 275.6 276.0 279.0 279.2 274.0 268.8 50.8 247.0 245.4 242.0 240.1 239.3 231.9 214.5 197.1

\$-----rear_downforce [REAR_DOWNFORCE] (Z_DATA) {rear_ride_height } 6.35 12.7 19.05 25.4 38.1 50.8 63.5 76.2 (XY_DATA) {front_ride_height downforce } 6.35 381.7 371.1 337.9 330.1 322.1 360.6 355.1 345.9 12.7 379.2 371.0 359.5 353.7 343.4 333.9 324.0 314.3 370.2 19.05 379.8 358.1 352.2 341.9 333.1 324.1 315.3 25.4 379.9 368.5 356.1 350.0 339.6 331.0 322.6 314.1 31.75 382.4 371.4 358.3 351.8 341.1 331.2 319.9 308.6 38.1 384.1 373.3 359.1 352.0 340.9 328.9 313.2 297.5 50.8 385.9 375.3 359.9 352.4 340.7 326.4 306.4 286.5 \$-----drag [DRAG] (Z_DATA) {rear_ride_height } 6.35 12.7 19.05 25.4 38.1 50.8 63.5 76.2 (XY_DATA) {front_ride_height drag} 6.35 402.7 405.8 408.9 410.4 413.5 416.6 419.7 422.8 380.8 387.5 393.6 399.6 12.7 373.6 380.2 381.0 382.9 19.05 362.9 371.4 373.9 365.8 365.5 365.4 366.5 369.0 25.4 352.3 351.5 350.5 350.0 350.3 354.2 360.7 367.1 31.75 340.2 337.6 333.8 331.8 329.9 328.8 326.7 324.5 38.1 328.3 324.0 317.3 314.0 309.9 304.0 293.6 283.1 50.8 316.5 310.4 301.0 296.2 290.0 279.2 260.5 241.8

5.3. EV battery equation and application to motor torque

Header Driveline				
Driveline	e Layout Motors	Battery	Differentials	
Batte	ry			
Ти			Lision	_]
	Itane [V]			
	Constant			
	Function of SOC	Pla	t/Edit Data	
Pol	arizationConstant [\	/ / Ah]	0.0014563	
Ma	ximum Battery Capa	acity [Ah]	72.0	
Re	sponse Time		10.0	
Re	sistance [Ohm]		0.0125	
Sta	te of Charge [%]		25.0	
Exp	oonential Voltage [V		3.9883	
Exp	oonential Capacity [1/Ah]	1.6667	
Ma	x Peak Current [A]		1500.0	

Experimental Validation of a Battery Dynamic Model for EV Applications Olivier Tremblay1, Louis-A. <u>Dessaint, Electrical</u> Engineering <u>Department</u>, <u>E'cole</u> de Technologie <u>Supe'rieure</u>

 $V_{batt} = \text{battery voltage } (V)$ $E_0 = \text{battery constant voltage } (V)$ $K = \text{polarisation constant } (V/(Ah)) \text{ or polarisation resistance } (\Omega)$ Q = battery capacity (Ah) $it = \int idt = \text{actual battery charge } (Ah)$ A = exponential zone amplitude (V) $B = \text{exponential zone time constant inverse } (Ah)^{-1}$ $R = \text{internal resistance } (\Omega)$ i = battery current (A)

Figure 4: Battery model reference and variables

- PolarizationConstant [V / Ah] Battery polarization constant.
- Maximum Battery Capacity [Ah] Battery maximum capacity.
- Response Time
 Battery response time.
- Resistance [Ohm] Battery resistance.
- State of Charge [%] Battery initial state of charge.
- Exponential Voltage [V] Battery exponential voltage.
- Exponential Capacity [1/Ah] Battery exponential capacity.
- Max Peak Current [A]
 Battery maximum peak current.

 V_{batt} = battery voltage (V)

- $$\begin{split} E_0 = & \text{battery constant voltage } (V) \\ K = & \text{polarisation constant } (V/(Ah)) \text{ or polarisation resistance } (\Omega) \end{split}$$
- Q =battery capacity (Ah)
- $it = \int idt$ = actual battery charge (Ah)
- A = exponential zone amplitude (V)
- B = exponential zone time constant inverse $(Ah)^{-1}$
- $R = internal resistance (\Omega)$
- i =battery current (A)

 $i^* =$ filtered current (A)

- Lead-Acid Discharge : $V_{batt} = E_0 - R \cdot i - K \frac{Q}{Q - it} \cdot (it + i^*) + Exp(t)$

 $\begin{array}{l} \text{Charge}: V_{batt} = E_0 - R \cdot i - K \frac{Q}{it - 0.1 \cdot Q} \cdot i^* - \\ K \frac{Q}{Q - it} \cdot it + Exp(t) \end{array}$

- Liion Discharge : $V_{batt} = E_0 - R \cdot i - K \frac{Q}{Q - it} \cdot (it + i^*) + Aexp(-B \cdot it)$

 $\begin{array}{l} \text{Charge}: V_{batt} = E_0 - R \cdot i - K \frac{Q}{it - 0.1 \cdot Q} \cdot i^* - \\ K \frac{Q}{O - it} \cdot it + Aexp(-B \cdot it) \end{array}$

• Ni-Cd Discharge : $V_{batt} = E_0 - R \cdot i - K \frac{Q}{Q - it} \cdot (it + i^*) + Exp(t)$

$$\begin{split} \text{Charge}: V_{batt} &= E_0 - R \cdot i - K \frac{Q}{|it| - 0.1 \cdot Q} \cdot i^* - \\ K \frac{Q}{Q - it} \cdot it + Exp(t) \end{split}$$

• Ni-Mh

Discharge : $V_{batt} = E_0 - R \cdot i - K \frac{Q}{Q - it} \cdot (it + i^*) + Exp(t)$

$$\begin{split} \text{Charge}: V_{batt} = E_0 - R \cdot i - K \frac{Q}{|it| - 0.1 \cdot Q} \cdot i^* - \\ K \frac{Q}{Q - it} \cdot it + Exp(t) \end{split}$$

Figure 5: Battery model primary equations

> VICRT limits motor torque such that motor power is less than battery power

Torque in SI units can be calculated as

 $\begin{array}{l} \underline{T_{Nm}} = P_W \, 9.549 \, / \, n \\ \underline{where} \\ \underline{T_{Nm}} = torque \, (Nm) \\ P_W = \, power \, (watts) \\ n = revolution \, per \, minute \, (rpm) \end{array}$

> Is less than batter power



 $P_{\rm W}$ (watts) is less than battery voltage * current ^ 2

Figure 6: Motor torque limited by Battery power